L Number	Hits	Search Text	DB	Time stamp
1	56	375.clas. and ofdm and (isi with channel) and zero and (cyclic adj prefix)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/22 09:47
2	0	prefix near2 cyclic and ((scalar$ near vector$) near4 (convert$3))	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/22 09:48
3	0	ofdm and prefix near2 cyclic and (scalar$ near vector$)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/22 09:49
4	16	ofdm and prefix near2 cyclic and (scalar$ same vector$)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/22 09:49
5	10	ofdm and prefix near2 cyclic and (scalar$ same vector$) and zero	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/22 09:51
-	5	bocure.xp. and (cyclic adj prefix)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 13:48
-	1	(bocure.xp. and (cyclic adj prefix)) and "ofdm"	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 13:49
-	268	"ofdm" same (cyclic with prefix)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 13:53
-	252	("ofdm" same (cyclic with prefix)) and symbol	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 13:52
-	148	("ofdm" same (cyclic with prefix)) and symbol and vector	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 13:53
-	31	"ofdm" same (cyclic with prefix) same symbol same vector	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/02/19 14:46
-	47	"ofdm" and (cyclic with prefix) same symbol same vector	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 15:05
-	373	"ofdm" and (cyclic with prefix)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 15:05
-	166	"ofdm" and (cyclic with prefix) and 370.clas.	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 15:52
-	83	370.clas. and ofdm and isi and channel and zero	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 15:53
-	53	370.clas. and ofdm and (isi with channel) and zero	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 15:53
-	32	370.clas. and ofdm and (isi with channel) and zero and (cyclic adj prefix)	USPAT; US-PGPUB; EPO; JPO; DERWENT	2004/03/19 16:18

Precoder for DMT with insufficient cyclic prefix

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Index Terms:
interference suppression modulation coding subscriber loops transient response t ADSL DMT IIR filter QAM VDSL asymmetric digital subscriber lines channel o prefix length discrete multi-tone distortion high transmission rate impulse respons interference line impulse response multi-carrier modulation performance quadratu modulation transmitter very high speed DSL

Precoder for DMT with insufficient cyclic prefix

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Abstract

Multi-carrier modulation is a modulation technique that enables higher transmission rate than traditional techniques, such as single-carrier Quadrature Amplitude Modulation (QAM) systems. One implementation of multi-carrier modulation technique is Discrete Multi-tone (DMT). DMT used in Asymmetric Digital Subscriber Lines (ADSL) and Very High speed DSL (VDSL) systems has a fixed cyclic prefix length. When the impulse response of the channel is longer than the designed prefix length, for instance when the length of the line impulse response is very long, distortion will appear at the channel output in the form of interference between the carriers. This paper describes a method to remove this distortion by introducing a precoder at the transmitter. Although there will be a power increase when the precoder is used, the distortion is significantly reduced, yielding a performance improvement.

1 Introduction

Multi-carrier modulation (MCM) is increasingly employed on channels that have a high variation in attenuation across the bandwidth used [1]. It provides flexibility in using the carriers to adapt to different channel environments by adjusting the energy and constellation size of each of the carriers. One such implementation of MCM is the Discrete Multi-tone (DMT) system. In this system, a cyclic prefix is required in order to partition the channels into independent sub-carriers. The cyclic prefix has to be as long as the channel impulse response. However, in practical designs, the cyclic prefix is usually fixed. As a result, distortion might occur at the channel output if the channel impulse response is longer than the cyclic prefix. The distortion may be so severe that it dominates other noise.

This paper describes a method to overcome the distortion caused by insufficient cyclic prefix length. A precoder is used at the transmitter to ensure that distortion does not exist at the receiver. Section 2 describes the problem that could arise in DMT. Section 3 introduces the precoder and shows how the distortion can be removed a the expense of minor increase in transmit power. Section 4 shows the performance of this precoder.

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2 Discrete Multi-tone

A typical DMT modulation system is shown in Figure 1. An inverse fast Fourier transform (IFFT) is used to modulate the input signals to create a set of parallel sub-channels. The number of sub-channels is dependent on the size of the IFFT. For example, if an N-point IFFT is used, there are $N/2$ complex tones. This is because the input to the IFFT is made to be Hermitian so that the output of the IFFT is real. N is typically a large number, for example, 512. During system start-up, the channel and noise environment are estimated so that a bit loading algorithm, for example Chow's algorithm [2], can assign the energy and the number of bits to be carried on each sub-channel. The bit loading algorithm approximates water-filling solution in calculating capacity of a channel.

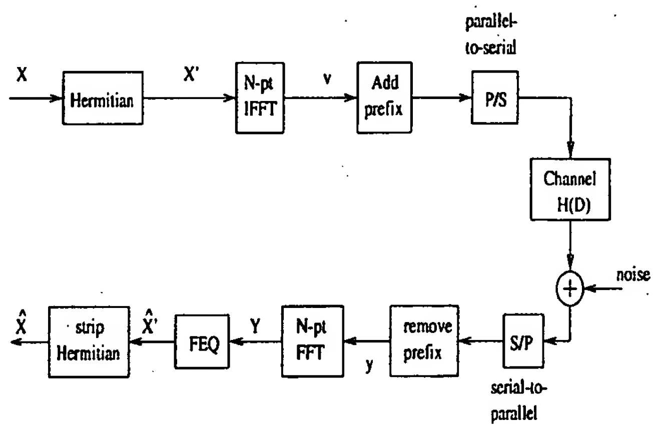

Figure 1: Discrete Multi-tone Modulation System.

Before the signals at the output of the IFFT are sent to the channel, a cyclic prefix is attached in order to ensure that the set of parallel sub-channels is independent. This can be seen as follows. If the channel is of length ν, so that the number of taps in the impulse response is $\nu + 1$, a cyclic prefix of length at least ν must be used. If this is the case, then the effect of the channel, which is a linear convolution of the signals with the channel impulse response, would appear to be a circular convolution. The channel can then be represented in the frequency domain as:

$$\mathbf{Y}_i = \Lambda_i \mathbf{X}_i + \mathbf{N}_i, \quad i = 0, \ldots, \frac{N}{2} - 1, \qquad (1)$$

where $\mathbf{Y}$ is the received signal, $\mathbf{X}$ is the transmitted signal, and $\mathbf{N}$ is the additive white Gaussian noise (AWGN). Λ_i is the i^{th} value of the N-point FFT of the channel impulse response, as will be seen in Section 3. The set of parallel channels is seen to be independent. However, if the cyclic prefix is not long enough, the channel will not be partitioned into set of parallel independent channel this way. Distortion will appear at the channel output.

Suppose that the prefix length is not long enough. Also, for simplicity, assume that the length of the channel impulse response is at most N. Denote the impulse response of the channel by $h = h_0 + h_1 D + \ldots + h_l D^l$, where $l \leq N$. We can write the channel output as follows:

$$\mathbf{y}_k = \tilde{H}\mathbf{v}_k + H_1\mathbf{v}_k + H_2\mathbf{v}_{k-1} + \mathbf{n}_k, \tag{2}$$

where $\tilde{H}$ is a $N \times N$ circular matrix with the channel impulse response as the first row,

$$\tilde{H} = \begin{bmatrix} h_0 & h_1 & \ldots & h_l & 0 & \ldots & 0 \\ 0 & h_0 & \ldots & h_{l-1} & h_l & \ldots & 0 \\ \vdots & & \ddots & & & & \vdots \\ h_l & 0 & \ldots & h_0 & h_1 & \ldots & h_{l-1} \\ h_{l-1} & h_l & \ldots & 0 & h_0 & \ldots & h_{l-2} \\ \vdots & & \ddots & & & \ddots & \vdots \\ h_1 & h_2 & \ldots & h_l & 0 & \ldots & h_0 \end{bmatrix}, \tag{3}$$

H_1 and H_2 are given by:

$$H_1 = \begin{bmatrix} \mathbf{0}_{(N-l+\nu)\times\nu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}_{(l-\nu)\times\nu} & -H_t & \mathbf{0}_{(l-\nu)\times(N-l)} \end{bmatrix}, \tag{4}$$

$$H_2 = \begin{bmatrix} \mathbf{0}_{(N-l+\nu)\times(l-\nu)} & \mathbf{0}_{(N-l+\nu)\times(N-l+\nu)} \\ H_t & \mathbf{0} \end{bmatrix}, \tag{5}$$

where,

$$H_t = \begin{bmatrix} p_l & 0 & \ldots & 0 \\ p_{l-1} & p_l & \ldots & 0 \\ \vdots & & \ddots & \vdots \\ p_{\nu+1} & p_{\nu+2} & \ldots & p_l \end{bmatrix}. \tag{6}$$

If the prefix is long enough, the channel would appear to be circular and H_1 and H_2 would both be $\mathbf{0}$ in Equation (2). H_1 and H_2 account for the inter-channel interference and the inter-block interference respectively. If nothing is done to correct the insufficient prefix length problem, at the receiver, the estimate of the signal sent is

$$\begin{aligned} \hat{\mathbf{X}}_k &= \Lambda^{-1}Q^*\mathbf{y}_k \\ &= \mathbf{X}_k + \Lambda^{-1}(Q^*H_1\mathbf{X'}_k + Q^*H_2\mathbf{X'}_{k-1} + Q^*\mathbf{n}_k), \end{aligned} \tag{7}$$

where $\Lambda = \mathrm{diag}[\Lambda_1, \ldots, \Lambda_{N/2}]$. Q is the N-point IFFT matrix,

$$Q = \frac{1}{\sqrt{N}} \begin{bmatrix} e^{-j2\pi\frac{N^2}{N}} & e^{-j2\pi\frac{N(N-1)}{N}} & \ldots & e^{-j2\pi\frac{N\cdot 0}{N}} \\ e^{-j2\pi\frac{(N-1)N}{N}} & e^{-j2\pi\frac{(N-1)^2}{N}} & \ldots & e^{-j2\pi\frac{(N-1)\cdot 0}{N}} \\ \vdots & & \ddots & \vdots \\ e^{-j2\pi\frac{0\cdot N}{N}} & e^{-j2\pi\frac{0\cdot(N-1)}{N}} & \ldots & e^{-j2\pi\frac{0\cdot 0}{N}} \end{bmatrix}, \tag{8}$$

and therefore Q^* is the N-point FFT matrix [3]. Hence the additional distortion is $\Lambda^{-1}(Q^*H_1\mathbf{X'}_k + Q^*H_2\mathbf{X'}_{k-1})$. If the channel impulse response has significant energy outside the prefix length, this distortion can be significant, as will be seen in Section 3.

There are many ways to combat this problem. One way is to increase the cyclic prefix length so that most channels would be covered with the increased cyclic prefix length. But this would increase the overhead in the system, hence decreasing the throughput. Increasing N would decrease the overhead, but this comes at the cost of increased delay, complexity and memory. Increased delay might be unacceptable for applications using the transmission system. Another way to mitigate the problem is to use a time-domain equalizer (TEQ) at the receiver to shorten the length of the channel to fit the pre-designed cyclic prefix length [4, 5]. Note that TEQ only partially equalizes the channel. Full equalization is not desired since it would lead to noise enhancement at the receiver. There are different ways to obtain the TEQ taps, and the optimization criterion is usually non-convex [4]. Non-convex optimization problems are difficult to solve and the solution obtained is not guaranteed to be the global minimum. A new scheme is proposed to overcome the distortion due to insufficient prefix length. This is introduced in the next section.

3 Precoder

Precoder has been proposed in many systems. Tomlinson-Harashima precoder [6, 7] is used in QAM to avoid the problem of error propagation in the decision-feedback equalizers (DFE) at the receiver. The precoder causes a power increase of $M^2/(M^2-1)$ in general, where M is the number of levels in a PAM signaling. Multi-channel precoding is used in vector coding to remove error propagation in block DFE [8, 9]. The situation is different in DMT. It would be difficult to introduce a DFE at the receiver in DMT systems since the interference includes the present symbol as well as the previous symbol. The precoder for DMT eliminates the distortion by processing the signals at the transmitter so that the signals appear to be undistorted at the receiver. The basic scheme of the precoder used is shown in Figure 2. As in any modulation technique that requires bit loading, the channel transfer function $H(D)$ is known to both the transmitter and receiver. We also assume that $H(D)$ is stable and has no zeros on the unit circle, $\{D : \|D\| = 1\}$. $\Gamma_{\mathbf{M}}(\cdot)$ is the modulo operator. It acts on the input as follows:

$$\{\Gamma_{\mathbf{M}}(\mathbf{X})\}_i = \mathbf{X}_i - \mathbf{M}_i d_i \left\lfloor \frac{\mathbf{X}_i + \frac{\mathbf{M}_i d_i}{2}}{\mathbf{M}_i d_i} \right\rfloor, \quad i = 1, \ldots, \frac{N}{2}, \tag{9}$$

where $\mathbf{M}_i$ is the number of bits assigned to the i^{th} sub-channel and d_i is the minimum distance between constellation points in the i^{th} sub-channel. It can be shown that the range of $\{\Gamma_{\mathbf{M}}(\cdot)\}_i$ is $[-\mathbf{M}_i d_i/2, \mathbf{M}_i d_i/2)$. W and B are block

filters. The Frequency Domain Equalizer (FEQ) is simply a vector of complex numbers that adjust the phase and magnitude of the output of the FFT so that a common decision element can be used for the signals in all the carriers. We can represent the FEQ with Λ^{-1}.

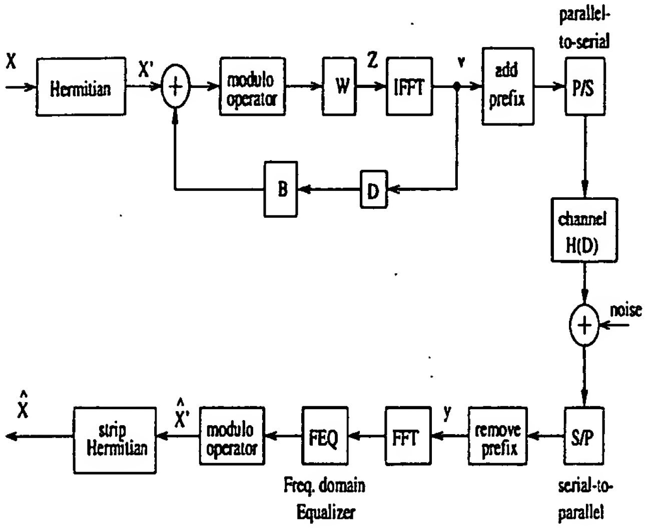

Figure 2: Precoder.

We briefly describe the precoder and show how it removes distortion caused by insufficient cyclic prefix length. For simplicity, we assume that the channel impulse response is not longer than N, where N is the number of sub-channels. From Figure 2, we have

$$W^{-1}\mathbf{Z}_k = \Gamma_{\mathbf{M}}(\mathbf{X'}_k + B\mathbf{v}_{k-1}), \qquad (10)$$

where $\mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_{N/2}, \mathbf{M}_{N/2}, \dots, \mathbf{M}_1]^T$ and the subscript k in the equation refers to the index for the signal blocks. Also, $\mathbf{v}_k = Q\mathbf{Z}_k$, where Q is the N-point IFFT matrix. At the receiver, we have

$$\hat{\mathbf{X}}_k = \Gamma_{\mathbf{M}}\left(\Lambda^{-1}Q^*\mathbf{y}_k\right), \qquad (11)$$

We can write $\mathbf{y}_k = \tilde{H}\mathbf{v}_k + H_1\mathbf{v}_k + H_2\mathbf{v}_{k-1}$, as in (2). Note that Q diagonalizes $\tilde{H}$, with $\tilde{H} = Q\Lambda Q^*$ and the diagonal of Λ is the FFT of the first row of $\tilde{H}$. Neglecting additive noise, and without going through the algebra here, if we choose W and B such that

$$W = [I + Q^*\tilde{H}^{-1}H_1Q]^{-1}, \qquad (12)$$

$$B = -Q^*\tilde{H}^{-1}H_2, \qquad (13)$$

we have $\hat{\mathbf{X}}_k = \mathbf{X}_k$. Hence all distortion due to insufficient cyclic prefix is removed. However, the cost of this benefit is a small increase in transmit power. This increase in power comes from the modulo operator $\Gamma_{\mathbf{M}}$ that is required at the transmitter to limit the transmit power. Although an upper bound for this power increase can be calculated, it is found that this bound is seldom reached. Instead, the following simulation results obtain the power increase through simulations. Note also that because of the matrix multiplies, we have $O(N^2)$ complexity for the precoder. Since H_1 and H_2 are usually sparse matrices, the complexity can be reduced. Also, we could approximate the solution for W and B so that we can implement them with less complexity, although this would introduce distortion at the channel output.

4 Performance

This section shows two simulation results for the precoder. The result from the precoder are compared to the ideal case where all distortion can be removed without any penalty (upper bound for performance) and the case where the distortion is left as it is. In the following simulations, the channels used are truncated to N taps for simplicity. Only AWGN is considered in the simulation.

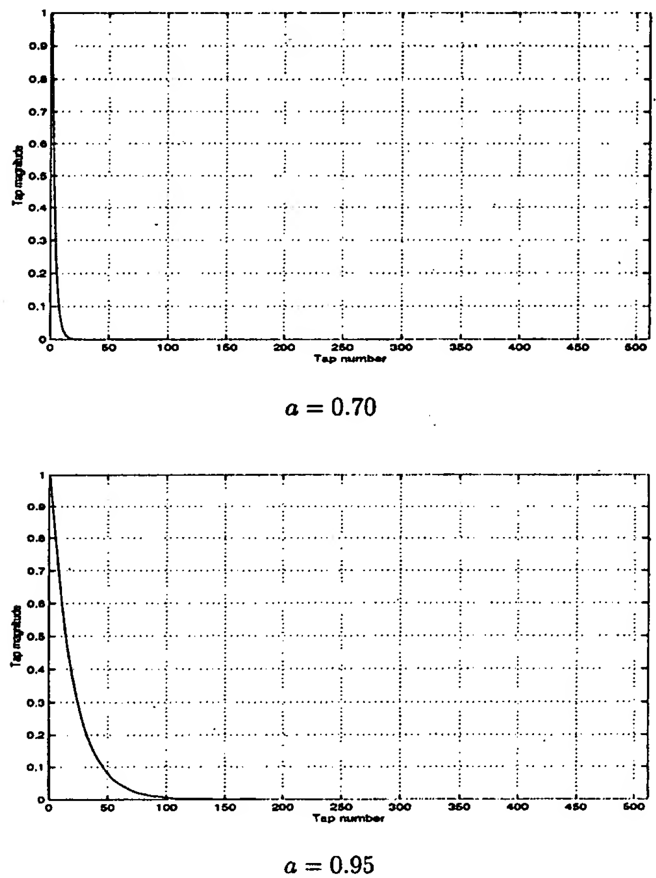

Figure 3: Channel impulse responses for first simulation.

In the first simulation, the channel used is a truncated single pole IIR channel, $H(D) = \frac{1}{1+aD}$, where $0 \le a < 1$. This channel is used to illustrate the distortion that will appear at the channel output when the cyclic prefix is not long enough. When $a = 0$, we have a 1 tap channel. As $a \to 1$, the channel impulse response has significant energy

outside the cyclic prefix length. See Figure 3. We expect that for large a, there will be significant distortion at the channel output. The number of channels, N, is taken to be 256 and the prefix length, ν, is set at 16. This gives an overhead of 6.25%, which is reasonable. The noise level is such that the matched-filter bound SNR, $\mathrm{SNR_{MFB}}$, is 30 dB. $\mathrm{SNR_{MFB}}$ is defined as the ratio of input energy multiply by the energy of the channel impulse response to the noise. The target probability of error, P_e, is 10^{-6}. The result of the simulation is shown in Figure 4. From the figure, we see that distortion at the channel output caused by insufficient prefix length can be much larger than other noise present in the system.

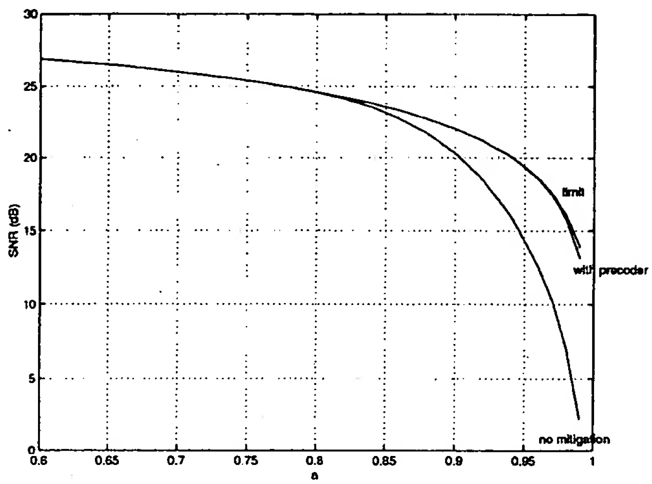

Figure 4: Simulation result for $N = 256, \nu = 16$.

In the second simulation, we pick a model of a twisted pair shown in Figure 5. The twisted pair consist of a long line of x ft. and bridge taps at the end of the line. The length of the line is variable. As the length of the line increases, the channel becomes more disperse. In this simulation, the number of channels, N, is 512 and the prefix length, ν, is 40, giving an overhead of 7.8%. The number of taps in the channel impulse response depends on the sampling rate, which is chosen such that $1/T = 22.08$ MHz. An example of the channel impulse response is shown in Figure 6. The length of the line is 1500 ft. For short enough length, the impulse response of the line is usually within the cyclic prefix length, but it would have significant energy outside the prefix length if the length of the line is long, as is shown in the figure. Average signal power is taken to be -60 dBm/Hz and additive white Gaussian noise is -130 dBm/Hz. The target probability of error is again $P_e = 10^{-6}$. The result of the simulation is shown in Figure 7.

From the simulations, we see that the DMT with the precoder is able to reduce the performance degradation due to insufficient cyclic prefix length. The upper curve represents the SNR when all the distortion can be removed with no penalty due to transmit energy increase and the lower curve represents the SNR when nothing is done to remove

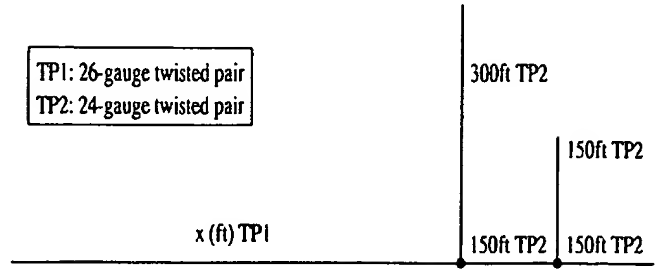

Figure 5: Twisted pair configuration.

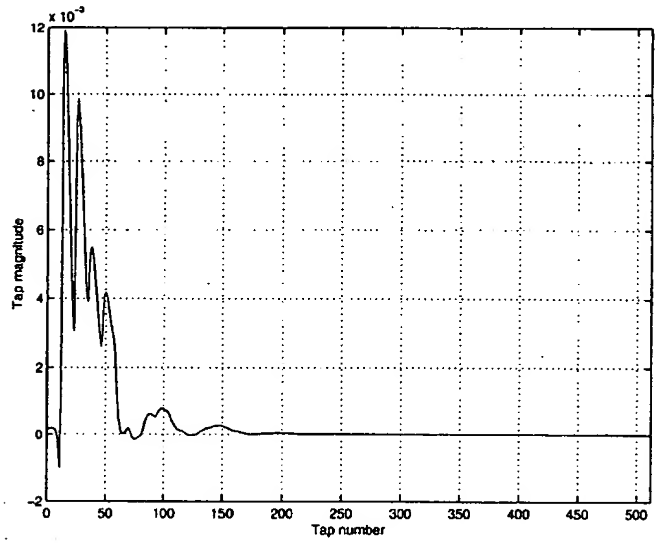

Figure 6: Channel taps for twisted pair, length=1500 ft.

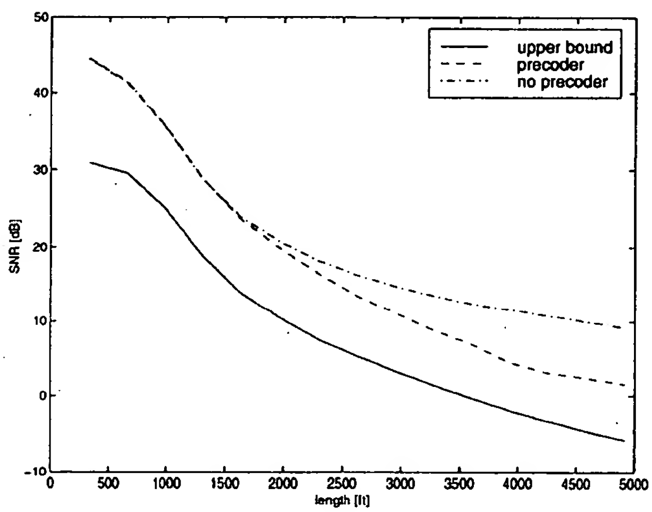

Figure 7: Simulation result for twisted pair in Figure 5.

the distortion. We see that the energy increase is minimal for the precoder. The penalty in performance due the power increase is minimal for the precoder. The reason for this is the feedback portion of the precoder has taps that are small. Hence, at the output of the modulator, the signals are not exactly uniformly distributed in the region $[-M_i d/2, M_i d/2)$, for $i = 1, \dots, N/2$, as would be the case if the feedback taps are significant.

5 Conclusion

In this paper, we discuss the distortion at the channel output when insufficient cyclic prefix length is used for DMT systems. We propose to use a precoder at the transmitter to remove the distortion due to insufficient cyclic prefix length. Simulations using a single pole IIR filter and a typical telephone line with VDSL signaling rate as channel impulse response are done. It is demonstrated that the method produces significant improvement over when nothing is done to mitigate the distortion, even though there is a power penalty, which is shown to be minimal.

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